Name solutions

ECE 311 Exam 3 Fall 2009

December 1, 2009

Closed Text and Notes

1) Be sure you have 10 pages and the additional 5 pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

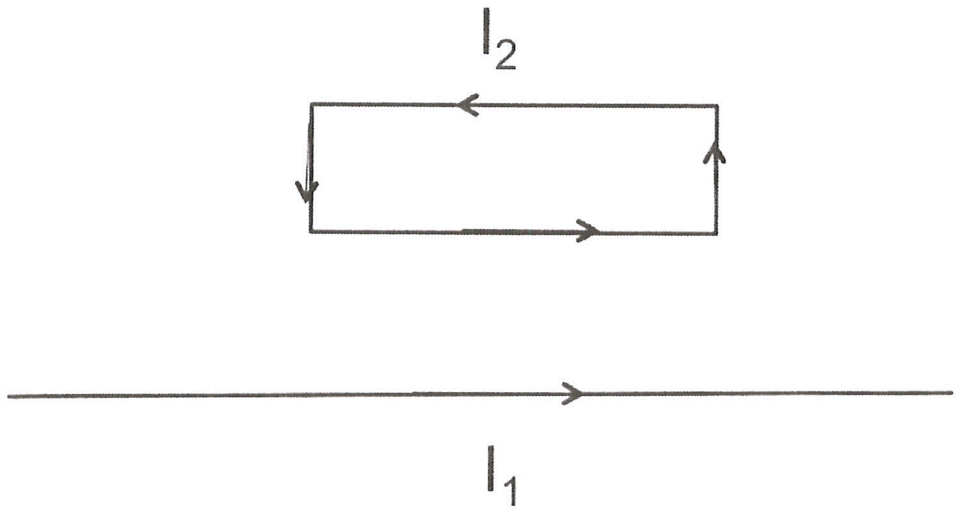
3) Write neatly, if your writing is illegible then print.

4) This exam is worth 100 points.

(8 pts) 1. Circle True or False concerning the following statements about the electric force $\mathbf{F_e}$ and the magnetic force $\mathbf{F_m}$ on a charged particle.

$\mathbf{E}$ and $\mathbf{F_e}$ are parallel to each other, whereas $\mathbf{B}$ and $\mathbf{F_m}$ are perpendicular to each other	[✓ True]	False
Both $\mathbf{F_e}$ and $\mathbf{F_m}$ depends on the velocity of the charged particle	True	[✓ False]
Both $\mathbf{F_e}$ and $\mathbf{F_m}$ can perform work.	True	[✓ False]
You cannot have $\mathbf{F_e}$ without $\mathbf{F_m}$	True	[✓ False]

(5 pts) 2. A wire loop carries current I_2, and is located near an infinite wire carrying current I_1. The currents flow in the directions shown.



The net force on the wire loop due to the presence of the infinite wire is

A) upwards
[✓ B)] downwards
C) to the left
D) to the right
E) no movement since there is no net force on the loop

(5 pts) 3. A magnetic material has 10^{29} atoms/m^3, each with a magnetic dipole moment of 10^{-27} Am^2 $\mathbf{a}_x$. What is the value of **M** for this material, with appropriate units?

$$\vec{M} = \frac{\text{dipole moment}}{\text{unit volume}}$$

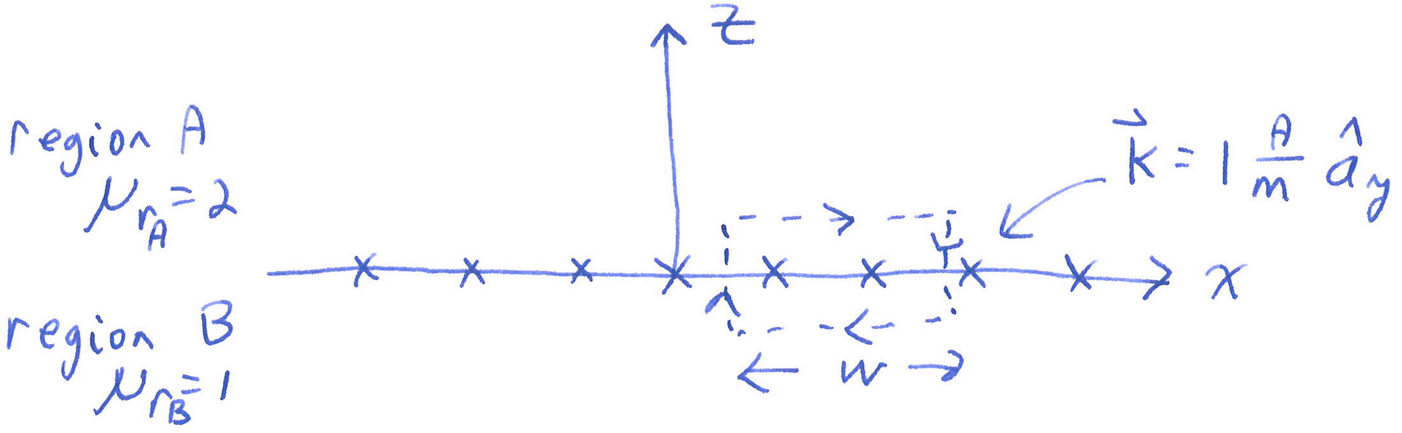
$$= \left(10^{29}\frac{\text{atoms}}{m^3}\right)\left(10^{-27}\frac{Am^2}{\text{atom}}\,\hat{a}_x\right)$$

$$\vec{M} = 100\,\frac{A}{m}\,\hat{a}_x$$

(8 pts) 4. Circle true or false concerning the statements for a ferromagnetic material.

They are capable of being magnetized very strongly by a magnetic field.	[✓ True]	False
They retain a considerable amount of their magnetization when removed from the field.	[✓ True]	False
There is a linear relationship between B and H.	True	[✓ False]
They loose their ferromagnetic properties when cooled below the Curie temperature.	True	[✓ False]

(10 pts) 5. The relative permeability is 1 for z < 0 and 2 for z > 0. In the xy plane there is a sheet current density of $\mathbf{K}=1\frac{\text{A}}{\text{m}}\hat{\mathbf{a}}_y$. If $\mathbf{H}=(1\hat{\mathbf{a}}_x+1\hat{\mathbf{a}}_z)\frac{\text{A}}{\text{m}}$ for z < 0, what is **H** for z > 0?



$\vec{H}_{Bn} = 1\,\hat{a}_z\,\frac{A}{m}$

$\vec{B}_{Bn} = \vec{B}_{An}$

$\mu_{rB}\mu_o\vec{H}_{Bn} = \mu_{ra}\mu_o\vec{H}_{An}$

$\vec{H}_{An} = \frac{\mu_{rB}}{\mu_{rA}}\vec{H}_{Bn}$

$= \frac{1}{2}\left(1\,\hat{a}_z\frac{A}{m}\right)$

$= \frac{1}{2}\hat{a}_z\frac{A}{m}$

$\vec{H}_{Bt} = 1\,\hat{a}_x\frac{A}{m}$

$\oint \vec{H}\cdot d\vec{l} = I_{encl.} = \vec{K}w$

↑ over the dashed path shown

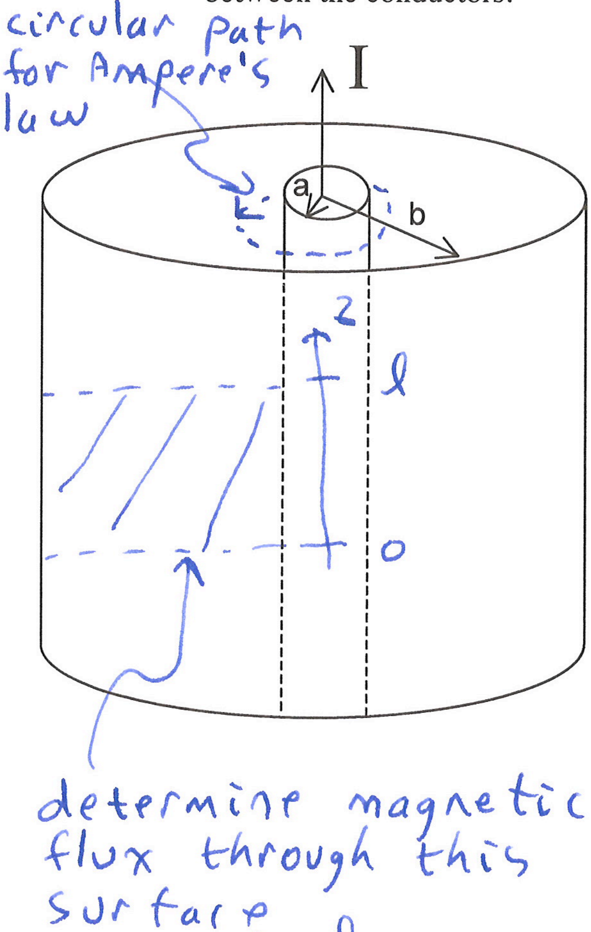
$H_{At}w - H_{Bt}w = Kw$

$H_{At} - 1\frac{A}{m} = 1\frac{A}{m}$

$\vec{H}_{At} = 2\frac{A}{m}\hat{a}_x$

$$\vec{H}_A = \left(2\hat{a}_x + \frac{1}{2}\hat{a}_z\right)\frac{A}{m}$$

(15 pts) 6. Derive the inductance per unit length for the coaxial cable shown. The inner conductor has radius a and the outer conductor radius b. The outer conductor is grounded. Assume free space between the conductors.



First find $\vec{H}$ for the coaxial cable using Ampere's Law

$$\oint \vec{H}\cdot d\vec{l} = I_{enclosed}$$

$$\int_0^{2\pi} H_\phi \hat{a}_\phi \cdot \rho\, d\phi\, \hat{a}_\phi = I$$

$$2\pi\rho H_\phi = I$$

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \,, \quad a<\rho<b$$

$$\Psi = \int_a^b\int_0^l \vec{B}\cdot dz\, d\rho\, \hat{a}_\phi = \int_a^b\int_0^l \frac{\mu_0 I}{2\pi\rho}\hat{a}_\phi \cdot dz\, d\rho\, \hat{a}_\phi$$

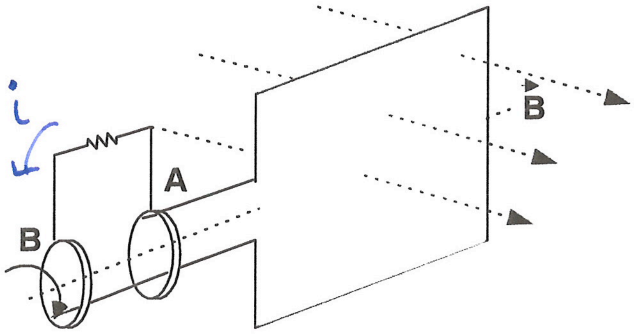
$$= \int_a^b\int_0^l \frac{\mu_0 I}{2\pi\rho} dz\, d\rho = \frac{\mu_0 I}{2\pi}\int_a^b\int_0^l dz \frac{d\rho}{\rho}$$

$$= \frac{\mu_0 I l}{2\pi}\int_a^b \frac{d\rho}{\rho} = \frac{\mu_0 I l}{2\pi}\ln\frac{b}{a}$$

$$L = \frac{\Psi}{I} = \frac{\mu_0 l}{2\pi}\ln\frac{b}{a}$$ ← inductance for section of coax of length l

$$\boxed{\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{b}{a}}$$

(15 pts) 7. A 1m by 1m wire loop is shown. This loop is rotated about the x-axis in a uniform magnetic field of $\mathbf{B}=1\hat{\mathbf{a}}_y\,\mathrm{T}$ at an angular frequency of $\omega=\frac{\pi}{60}\mathrm{s}^{-1}$. Determine the current flowing through the 100 Ω resistor shown. Assume the resistance of the wire loop is negligible and at t = 0 s the loop is perpendicular to the magnetic field intensity. In other words the figure shows the loop at t = 0 s.



let θ be the angle between the normal to the loop and the $\vec{B}$ field

$$\theta = \omega t = \frac{\pi}{60} t$$

$$\Psi = \vec{B} \cdot 1 m^2 \hat{a}_n = 1 \hat{a}_y \frac{Wb}{m^2} \cdot 1 m^2 \hat{a}_n$$

$$= 1 Wb \cos\theta = 1 Wb \cos\frac{\pi}{60} t$$

$$\frac{d\Psi}{dt} = -\frac{\pi}{60} \sin\frac{\pi}{60} t$$

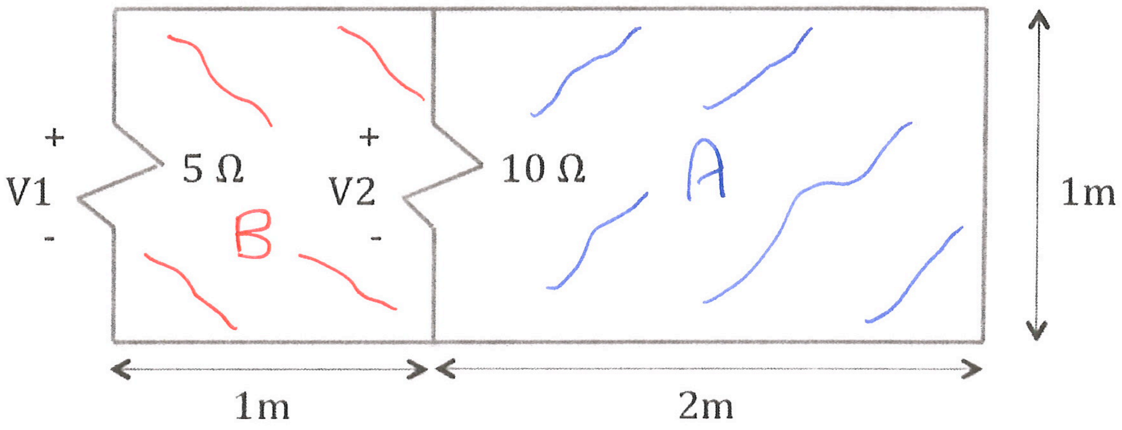
$$V_{emf} = -\frac{d\Psi}{dt}$$

$$i = \frac{V_{emf}}{100\,\Omega} = \frac{\pi}{60(100)} \sin\frac{\pi}{60} t \text{ A}$$

$$i = \frac{\pi}{6000} \sin\frac{\pi}{60} t \text{ A}$$

$$i = 5.23 \times 10^{-2} \sin\frac{\pi}{60} t \text{ A}$$

(12 pts) 8. The two-resistor circuit is in the field of magnitude $B=10t\frac{Wb}{m^2}$ that is into the page. Determine V1 and V2.



$$\oint \vec{E}\cdot\vec{dl} = V_{emf_A} = -\frac{d\psi}{dt} = \frac{d}{dt}\left[\left(10t\frac{Wb}{m^2}\right)2m^2\right]$$

ccw around A

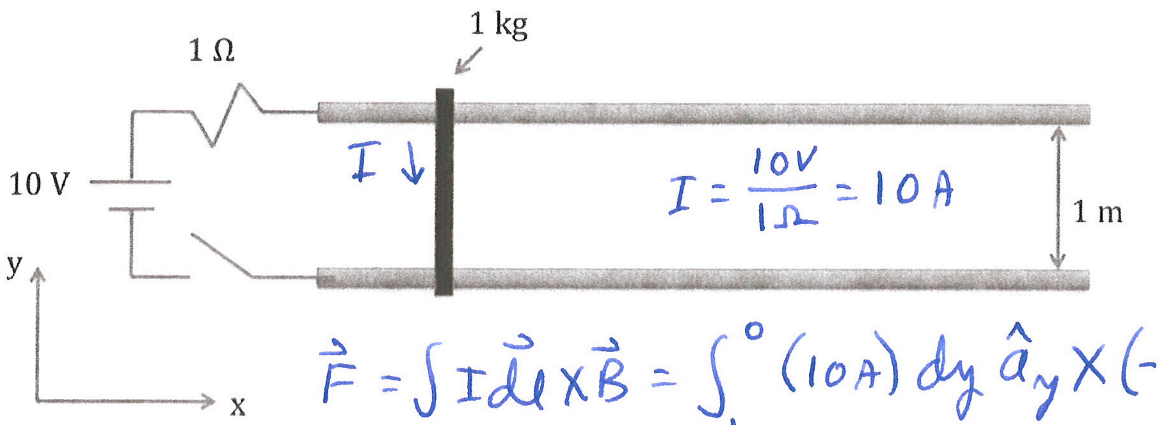
$$V_{emf_A} = 20V = V_2$$

$$\oint \vec{E}\cdot\vec{dl} = V_{emf_{A+B}} = -\frac{d\psi}{dt} = \frac{d}{dt}\left[\left(10t\frac{Wb}{m^2}\right)3m^2\right]$$

ccw around A+B

$$= 30V = V_1$$

(10 pts) 9. A 1 kg sliding bar is on the rail system shown. If the magnetic flux density is everywhere $\mathbf{B} = -5\hat{\mathbf{a}}_z\,\mathrm{T}$, what is the force on the sliding bar? Assume the resistance of the rails and the sliding bar are negligible.



$I = \frac{10V}{1\Omega} = 10A$

$$\vec{F} = \int I\,d\vec{l} \times \vec{B} = \int_{1m}^{0} (10A)\,dy\,\hat{a}_y \times \left(-5\hat{a}_z \frac{Wb}{m^2}\right)$$

$$\vec{F} = \left(-50A\frac{Wb}{m^2}\right)\int_{1m}^{0} dy\,\hat{a}_y \times \hat{a}_z = \left(-50A\frac{Wb}{m^2}\right)\left[\int_{1m}^{0} dy\right]\hat{a}_x$$

$$\vec{F} = 50\hat{a}_x\ A\frac{Wb}{m^2}m$$

$$\boxed{\vec{F} = 50\hat{a}_x\ N}$$

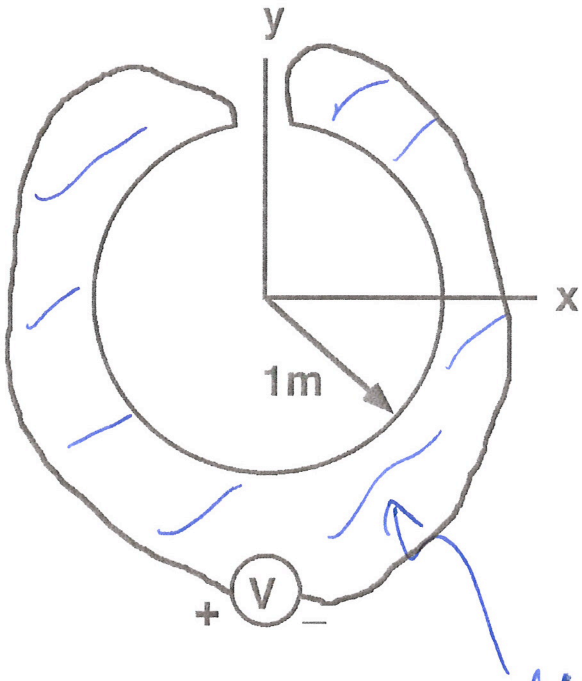
$$A\frac{Wb}{m^2}m = \frac{C}{s}\ \frac{V\text{-}s}{m} = \frac{CV}{m} = \frac{C\ J}{m\ C} = \frac{J}{m} = \frac{Nm}{m} = N$$

(5 pts) 10. A constant current of 10^{-3} A is charging a parallel plate capacitor. If the area of the capacitor plates is $10^{-3}\,\mathrm{m}^2$, what is the displacement current density between the capacitor plates?

$$I = \int \vec{J}_d \cdot d\vec{s} = J_d\,(10^{-3}m^2) = 10^{-3}A$$

$$J_d = 1\ \frac{A}{m^2}$$

(7 pts) 11. A magnetic field has a value of $\mathbf{B}=0$ for $\rho > 1\text{m}$ and $\mathbf{B}=10t\frac{\text{Wb}}{\text{m}^2}$ for $\rho < 1\text{m}$. A voltmeter is connected to a small gap in a circular conductor of radius 1 m as shown. The conductor is in the xy-plane. What is the reading on the voltmeter?



$\frac{d\Psi}{dt} = 0$ inside the circuit

so $V = 0$